Boundary σ model and corrections to D-brane actions

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We consider a σ-model formulation of open string theory in the presence of D-branes. We perform two-loop computations and discuss gravitational corrections to the Born-Infeld action when branes are nontrivially embedded in a curved ambient space. In particular, for the case of a stack of N coincident D-branes we analyze couplings of the form $R_{ijkl}[\Phi^i\Phi^j][\Phi^k\Phi^l]$.

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I. INTRODUCTION

To understand the dynamics of D-branes it is very important to study the low energy effective action. For bosonic strings to leading order in α' it is given by the Dirac-Born-Infeld (DBI) action [1,2]. For superstrings there is an additional Wess-Zumino term describing the coupling of a brane to Ramond-Ramond fields [3].[1] In this paper we discuss certain higher order corrections to the DBI action depending on the embedding of branes in a curved ambient space.

Suppose we have a Dp-brane nontrivially embedded in a target space. In the Einstein frame the effective action for a single D-brane is given by

$$S_{\rm DBI}=-\tau_p\int d\sigma^{p+1}e^{\Phi[-1-\gamma(p+1)/2]}\times\sqrt{-\det[\tilde{G}_{\alpha\beta}+e^{\gamma\Phi}(\tilde{B}_{\alpha\beta}+2\pi\alpha' F_{\alpha\beta})]}. \qquad (1)$$

Here $\gamma=-4/(D-2)$. We are using greek letters (μ, ν,...) for space-time coordinates, (α, β,...) for coordinates on the brane, and latin letters $(i,j,\dots)$ for coordinates transverse to the brane. Thus, in the case of a p-brane embedded in a D-dimensional target space, $\mu=0,...,D-1$, $\alpha=0,...,p$, and $i=p+1,...,D-1$. $\{\sigma^\alpha\}$ is a set of coordinates on the brane and the embedding in target space is given by $X^\mu(\sigma^\alpha)$. Most computations are done in a static gauge: $X^\alpha=\sigma^\alpha$, $X^i=X^i(\sigma^\alpha)$. The massless closed string fields $G_{\mu\nu}$, $B_{\mu\nu}$, and Φ are functions of X^μ and the massless open string field A^α is a function of σ^α. The action (1) describes the coupling of a brane to Neveu-Schwarz–Neveu-Schwarz (NS-NS) background bulk fields $G_{\mu\nu}$, $B_{\mu\nu}$, and Φ. The tilde denotes the induced quantities $\tilde{G}_{\alpha\beta}=G_{\mu\nu}(\partial X^\mu/\partial\sigma^\alpha)(\partial X^\nu/\partial\sigma^\beta)$, $\tilde{B}_{\alpha\beta}=B_{\mu\nu}(\partial X^\mu/\partial\sigma^\alpha)(\partial X^\nu/\partial\sigma^\beta)$. The action for the bulk massless fields is also well known. For example, in the bosonic case it is given to the leading order (in the Einstein frame) by

$$S_{\rm bulk}=\frac{1}{2k^2}\int d^Dx\sqrt{-G}\times\left(R-\frac{1}{12}e^{2\gamma\Phi}H_{\mu\nu\rho}H^{\mu\nu\rho}+\gamma\partial_\mu\Phi\partial^\mu\Phi\right), \qquad (2)$$

where $H_{\mu\nu\rho}=3\partial_{[\mu}B_{\nu\rho]}$. Subleading terms can be found in [9].

There are different ways to determine brane and bulk actions. One of them is to compute the string tree amplitudes for massless fields, expand them in powers of α', and look for terms in the effective action to reproduce them. In this way the α'^2 curvature corrections to the brane action in the superstring case [5] and the α' curvature corrections to the brane action for the bosonic string [6] were found. Another way is to compute the renormalization group beta function for the field theory of strings on the world-sheet. The consistency condition of (super)conformal invariance requires that these beta functions should vanish. Treating massless string fields as background, we get the equations of motion from which we can derive the effective action. It is believed that the two approaches are perturbatively equivalent (see for example [16,9]). In this paper we perform another check of the correspondence and also compute certain α' corrections to D-brane action evaluating the two-loop beta function in the sigma model.

In Sec. II we present the σ model relevant to the case of a single D-brane. Using the background field method, we compute the one-loop beta function for the scalar fields Φ_i describing transverse fluctuations of the brane in a curved ambient space. In Sec. III we reproduce some of the results of [6] in the σ-model language. In particular, we determine the coefficients of the possible $O(\alpha')$ curvature terms in the effective action and discuss field redefinitions.

The case of a stack of N coincident D-branes is somewhat more complicated. Now we expect the effective action of the brane to be generalized to a non-Abelian theory with gauge group U(N). In this case, fields get promoted to U(N) matrices. One of the open problems is how to expand the square root of the determinant since the ordering of the noncommutative fields is not clear. We are not addressing this problem here. In Sec. IV we describe the sigma model with boundary fermions and compute the one-loop beta function for the non-Abelian field A_α in a curved background. Setting the beta function to zero gives equations of motion consistent with the DBI action. It is interesting to study the higher order corrections to the DBI action. In the non-Abelian case in a curved background there is a new class of terms that could appear. In Sec. V we compute corrections of the form $R_{ijkl}[\Phi^i\Phi^j][\Phi^k\Phi^l]$ to the effective action in the bosonic and superstring cases. We discuss the results in Sec. VI.

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[1]For a review, see [4].

II. σ-MODEL ACTIONS

The σ-model action for a single Dp-brane in bosonic string theory contains two terms—bulk and boundary:

$$S_\Sigma=\frac{1}{4\pi\alpha'}\int_\Sigma d^2\sigma\sqrt{g}[g^{ab}G_{\mu\nu}(X)\partial_a X^\mu\partial_b X^\nu+i\epsilon^{ab}B_{\mu\nu}(X)\partial_a X^\mu\partial_b X^\nu+\alpha' R\Phi(X)], \tag{3}$$

$$S_{\partial\Sigma}=-i\int d\theta\left[\partial_\theta\sigma^\alpha A_\alpha+i\frac{1}{2\pi\alpha'}\,\partial_r X^\mu\Phi_\mu\right]. \tag{4}$$

Here, all the fields $G_{\mu\nu}$, $B_{\mu\nu}$, Φ, A_α, and Φ_i are functions of X^μ. Consider for the time being only terms involving $G_{\mu\nu}$ and Φ_i. They can also be rewritten as

$$S=\frac{1}{2\pi\alpha'}\int_\Sigma d^2z\,G_{\mu\nu}\partial X^\mu\bar\partial X^\nu+\frac{1}{2\pi\alpha'}\int_{\partial\Sigma}d\theta\,\partial_r X^i\Phi_i\,. \tag{5}$$

The last term was obtained using T duality from the σ model of open string theory without D-branes. We are going to use the background field method, expanding this action near a bare classical solution $X^\mu=\tilde X^\mu+\pi^\mu$. It is easier to use expansion in normal coordinates ξ^μ in space-time and ζ^α on the brane following [2,7]. For our purposes it is sufficient to expand to the fourth order in ξ and ζ:

$$\begin{aligned}S_\Sigma[X]=\frac{1}{2\pi\alpha'}\int d^2z\Big\{&G_{\mu\nu}\partial\tilde X^\mu\bar\partial\tilde X^\nu+G_{\mu\nu}(\partial\tilde X^\mu\bar\nabla\xi^\nu+\bar\partial\tilde X^\mu\nabla\xi^\nu)+R_{\mu\nu\rho\sigma}\partial\tilde X^\mu\bar\partial\tilde X^\sigma\xi^\nu\xi^\rho+G_{\mu\nu}\nabla\xi^\mu\bar\nabla\xi^\nu\\&+\frac{1}{3}D_\mu R_{\nu\rho\sigma\tau}\partial\tilde X^\nu\bar\partial\tilde X^\tau\xi^\mu\xi^\rho\xi^\sigma+\frac{2}{3}R_{\mu\nu\rho\sigma}(\partial\tilde X^\mu\bar\nabla\xi^\sigma+\bar\partial\tilde X^\mu\nabla\xi^\sigma)\xi^\nu\xi^\rho+\frac{1}{3}R_{\mu\nu\rho\sigma}\nabla\xi^\mu\bar\nabla\xi^\sigma\xi^\nu\xi^\rho\\&+\frac{1}{12}(D_\mu D_\nu R_{\rho\sigma\tau\omega}+4R^{\mu'}_{\mu\nu\rho}R_{\mu'\sigma\tau\omega})\partial\tilde X^\rho\bar\partial\tilde X^\omega\xi^\mu\xi^\nu\xi^\sigma\xi^\tau+\frac{1}{4}D_\mu R_{\nu\rho\sigma\tau}(\partial\tilde X^\nu\bar\nabla\xi^\tau+\bar\partial\tilde X^\nu\nabla\xi^\tau)\xi^\mu\xi^\rho\xi^\sigma\Big\}+\cdots,\end{aligned} \tag{6}$$

$$\begin{aligned}S_{\partial\Sigma}[X]=\frac{1}{2\pi\alpha'}\int d\theta\Big[&\partial_r\tilde X^i+\nabla_r\xi^i+\partial_r\tilde X^\lambda\left(\frac{1}{3}R^i_{\mu\nu\lambda}\xi^\mu\xi^\nu+\frac{1}{12}D_\mu R^i_{\nu\rho\lambda}\xi^\mu\xi^\nu\xi^\rho\right)+\left(\frac{1}{60}D_\mu D_\nu R^i_{\rho\sigma\lambda}-\frac{1}{45}R^i_{\mu\nu\tau}R^\tau_{\rho\sigma\lambda}\right)\xi^\mu\xi^\nu\xi^\rho\xi^\sigma\Big]\\&\times\left[\Phi_i+\tilde D_\alpha\Phi_i\zeta^\alpha+\frac{1}{2}\tilde D_\alpha\tilde D_\beta\Phi_i\zeta^\alpha\zeta^\beta+\frac{1}{6}\tilde D_\alpha\tilde D_\beta\tilde D_\gamma\Phi_i\zeta^\alpha\zeta^\beta\zeta^\gamma+\frac{1}{24}\tilde D_\alpha\tilde D_\beta\tilde D_\gamma\tilde D_\delta\Phi_i\zeta^\alpha\zeta^\beta\zeta^\gamma\zeta^\delta\right]+\cdots.\end{aligned} \tag{7}$$

Here D_μ is the usual covariant derivative with Levi-Civita connection, $\tilde D_\alpha$ is the covariant derivative on the brane constructed using the induced metric and

$$\nabla\xi^\mu=\partial\xi^\mu+\Gamma^\mu_{\nu\rho}\xi^\nu\partial\tilde X^\rho,\quad\bar\nabla\xi^\mu=\bar\partial\xi^\mu+\Gamma^\mu_{\nu\rho}\xi^\nu\bar\partial\tilde X^\rho,$$

$$\text{and}\ \nabla_r\xi^\mu=\partial_r\xi^\mu+\Gamma^\mu_{\nu\rho}\xi^\nu\partial_r\tilde X^\rho.$$

Using equations of motion we obtain an additional boundary term:

$$\frac{1}{2\pi\alpha'}\int d^2zG_{\mu\nu}(\partial X^\mu\bar\nabla\xi^\nu+\bar\partial X^\mu\nabla\xi^\nu)=\frac{1}{2\pi\alpha'}\int d\theta\partial_r X^\mu G_{\mu\nu}\xi^\nu.$$

In the bulk action, the kinetic term is multiplied by $G_{\mu\nu}(\tilde X)$. One way of bringing it into the standard form is to introduce the vielbein field V^A_μ, $G_{\mu\nu}=V^A_\mu V^B_\nu\eta_{AB}$, and switch to the tangent space quantities. Here we choose another approach. We expand the metric as $G_{\mu\nu}=\eta_{\mu\nu}+2kH_{\mu\nu}$ and expand the sigma model and effective actions in powers of H.

The "Neumann" and "Dirichlet" propagators on the disk are given, respectively, by

$$N^{\alpha\beta}(z,z')=\langle\xi^\alpha(z)\xi^\beta(z')\rangle=\frac{\alpha'}{2}\eta^{\alpha\beta}\{-\ln|z-z'|^2-\ln|1-\bar z z'|^2\}, \tag{8}$$

$$D^{ij}(z,z')=\langle\xi^i(z)\xi^j(z')\rangle=\frac{\alpha'}{2}\delta^{ij}\{-\ln|z-z'|^2+\ln|1-\bar z z'|^2\}. \tag{9}$$

Note that $D^{ij}(z,z')$ is zero if at least one of the variables is on the boundary. Thus we can neglect all transverse ξ^i's in the boundary terms (but not $\partial_r\xi^i$).

The superstring sigma model contains additional pieces with fermions. The supersymmetric extension of part of the bulk action containing the $G_{\mu\nu}$ field is given by

$$S_{\Sigma}=\frac{1}{2\pi\alpha'}\int_{\Sigma}d^2z\bigg\{G_{\mu\nu}\bigg(\partial\tilde{X}^{\mu}\bar{\partial}\tilde{X}^{\nu}+\frac{\alpha'}{2}\Psi^{\mu}\bar{\nabla}\Psi^{\nu}+\frac{\alpha'}{2}\bar{\Psi}^{\mu}\nabla\bar{\Psi}^{\nu}\bigg)+\frac{\alpha'^2}{2}R_{\mu\nu\rho\sigma}\Psi^{\mu}\Psi^{\nu}\bar{\Psi}^{\rho}\bar{\Psi}^{\sigma}\bigg\}. \quad (10)$$

The boundary action is obtained by T duality from the supersymmetric version of the Wilson loop. The pieces containing Φ_i fields are given by

$$S_{\partial\Sigma}=\frac{1}{2\pi\alpha'}\int_{\partial\Sigma}d\theta[\partial_r\tilde{X}^i\Phi_i+\alpha'(\psi^{\alpha}\bar{\psi}^{i}-\psi^{i}\bar{\psi}^{\alpha})\partial_{\alpha}\Phi_i]. \quad (11)$$

Here $\psi^{\mu}=\Psi^{\mu}|_{\partial\Sigma}$. We will not be interested in the supersymmetric extension of the term with the $B_{\mu\nu}$ field.

The expansion of the additional fermionic terms in normal coordinates sufficient for two-loop computation is

$$\frac{1}{4\pi}\int_{\Sigma}dz^2\bigg\{G_{\mu\nu}(\Psi^{\mu}\bar{\nabla}\Psi^{\nu}+\bar{\Psi}^{\mu}\nabla\bar{\Psi}^{\nu}) +R_{\mu\nu\rho\sigma}\bigg(\frac{1}{3}\xi^{\nu}\xi^{\rho}(\Psi^{\mu}\bar{\nabla}\Psi^{\sigma}+\bar{\Psi}^{\mu}\nabla\bar{\Psi}^{\sigma})+\alpha'\Psi^{\mu}\Psi^{\nu}\bar{\Psi}^{\rho}\bar{\Psi}^{\sigma} -\frac{1}{2}\bar{\partial}\tilde{X}^{\mu}\xi^{\nu}\Psi^{\rho}\Psi^{\sigma}-\frac{1}{2}\partial\tilde{X}^{\mu}\xi^{\nu}\bar{\Psi}^{\rho}\bar{\Psi}^{\sigma}\bigg)\bigg\} \quad (12)$$

and

$$\frac{1}{2\pi\alpha'}\int_{\partial\Sigma}d\theta\bigg[\alpha'(\psi^{\alpha}\bar{\psi}^{i}-\psi^{i}\bar{\psi}^{\alpha})\bigg(\partial_{\alpha}\Phi_i+\tilde{D}_{\beta}\partial_{\alpha}\Phi_i\zeta^{\beta} +\frac{1}{6}(3\tilde{D}_{\beta}\tilde{D}_{\gamma}\partial_{\alpha}\Phi_i+\tilde{R}^{\delta}_{\beta\gamma\alpha}\partial_{\delta}\Phi_i)\zeta^{\beta}\zeta^{\gamma}\bigg)\bigg]. \quad (13)$$

The fermionic propagators are

$$\langle\Psi^{\mu}(z)\Psi^{\nu}(w)\rangle=\frac{\eta^{\mu\nu}}{z-w},\quad \langle\Psi^{\mu}(z)\bar{\Psi}^{\nu}(w)\rangle=\frac{i\eta^{\mu\nu}}{1-z\bar{w}}. \quad (14)$$

Our strategy will be to compute the renormalization group beta function corresponding to the coupling $(1/2\pi\alpha')\int d\theta\,\partial_r X^i\Phi_i$. Following [10], we define the beta function for the field Φ_i to be

$$\beta_i^{\Phi}(\Phi^{\text{bare}})=-\frac{d}{d\ln\Lambda}\Phi_i^{\text{bare}}(\Phi). \quad (15)$$

Here,

$$\Phi_i^{\text{bare}}(\Phi)=\Phi_i+\sum_n K_i^{(n)}(\Phi)(\ln\Lambda)^n. \quad (16)$$

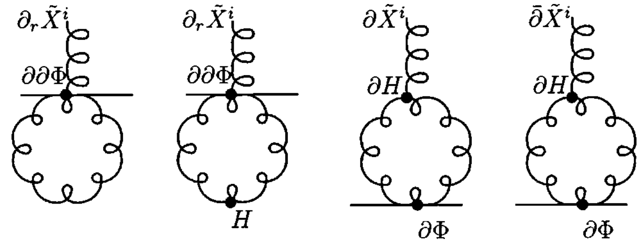

FIG. 1. One-loop corrections to the boundary vertex. Here horizontal lines represent the boundary and wiggly lines correspond to bosonic string coordinates.

Solving for Φ_i, one can obtain $\beta_i^{\Phi}(\Phi^{\text{bare}})$ in terms of Φ^{bare}. Now the explicit dependence on $\ln\Lambda$ must cancel, which is a good test of the calculations. Setting the beta function to zero will give us equations in $G_{\mu\nu}$ and Φ_i.

Let us compute the one-loop beta function to first order in $H_{\mu\nu}$ and Φ_i in the bosonic case. For simplicity, consider the case when $H_{\alpha i}=0$ and all fields depend only on σ^{α}. The general procedure is to leave normal coordinates $\partial_r\xi^i$ and ξ^{α} in the boundary action unchanged and express normal coordinates ξ^i (without derivative ∂_r) and ζ^{α} in the boundary action in terms of ξ^{α}. Under the above conditions this means: $\xi^i|_{\partial\Sigma}=0$ (but not $\partial_r\xi^i$) and $\xi^{\alpha}|_{\partial\Sigma}=\zeta^{\alpha}$. We have to evaluate the set of diagrams shown in Fig. 1. The last two diagrams with bulk vertices combine to produce a contribution proportional to $\partial_r\tilde{X}^i$. Their computation involves integration over d^2z and no integration over θ. The bulk integration can be done as follows [8]:

$$\int d^2z=\int r\,dr\int d\theta=\int r\,dr\oint\frac{dw}{iw}, \quad (17)$$

where $w=e^{i\theta}$. First one needs to perform the contour integration and then the integration over r, introducing a cutoff Λ [so that $N^{\alpha\beta}(u,u)=-\alpha'\eta^{\alpha\beta}\ln\Lambda^2$ for the point u on the boundary of the disk]. As a result, we get

$$\beta^{\Phi}{}_i=\alpha'\partial^2\Phi_i+2k\alpha'\bigg(-H_{\alpha\beta}\partial^{\alpha}\partial^{\beta}\Phi_i-\partial^{\alpha}H_{\alpha\beta}\partial^{\beta}\Phi_i +\frac{1}{2}\partial^{\alpha}H^{\beta}_{\beta}\partial_{\alpha}\Phi_i+\partial^{\alpha}H_{ij}\partial_{\alpha}\Phi_j\bigg). \quad (18)$$

It is easy to see that at this order the beta function is proportional to the equations of motion coming from

$$S_{\text{eff}}=-\tau_p\int d^{p+1}\sigma\sqrt{-\tilde{G}}, \quad (19)$$

where for the induced metric on the D-brane we have

$$\tilde{G}_{\alpha\beta}(\tilde{X}+\tilde{\Phi})=\eta_{\alpha\beta}+2kH_{\alpha\beta}+\partial_{\alpha}\tilde{\Phi}^i\partial_{\beta}\tilde{\Phi}^i+2kH_{ij}\partial_{\alpha}\tilde{\Phi}^i\partial_{\beta}\tilde{\Phi}^j \quad (20)$$

if we identify $\Phi_i\equiv\tilde{\Phi}^i$. More precisely, we get

$$\delta S_{\text{eff}}=\frac{\tau_p}{\alpha'}\int d^{p+1}\sigma\sqrt{-\tilde{G}}[\beta^{\Phi}(\tilde{\Phi})]^iG_{ij}\delta\tilde{\Phi}^j. \quad (21)$$

Computations in more general cases are also straightforward. For example, if we relax condition $H_{i\alpha}=0$ and allow $H_{\mu\nu}$ to depend on X^i then boundary term without Φ_i will also contribute to the beta function and we have

$$\xi^i|_{\partial\Sigma}=\tfrac{1}{2}\Gamma^i_{\alpha\beta}\xi^\alpha\xi^\beta+\tfrac{1}{6}\Gamma^i_{\alpha\beta\gamma}\xi^\alpha\xi^\beta\xi^\gamma+\tfrac{1}{24}\Gamma^i_{\alpha\beta\gamma\delta}\xi^\alpha\xi^\beta\xi^\gamma\xi^\delta + O(\xi^5).$$

Computations to higher orders in $H_{\mu\nu}$ and for $X^i(\sigma^\alpha)\neq$const become more tedious.

III. CURVATURE CORRECTIONS TO THE ACTION OF A SINGLE D-BRANE

In [6] the authors were able to determine first order curvature corrections to the Born-Infeld action in the case of a bosonic string by analyzing the tree level amplitudes corresponding to scattering of massless closed string fields off the brane. At order $O((\alpha')^0)$, they showed agreement between the string and field theory amplitudes using the expansion of the bulk and brane actions (2) and (1). The massless closed and open string fields are redefined as

$$G_{\mu\nu}=\eta_{\mu\nu}+2kH_{\mu\nu}, \quad \Phi=k\sqrt{(D-2)/4}\phi,$$

$$B_{\mu\nu}=-2kb_{\mu\nu},$$

$$\tilde{\Phi}^i=\frac{1}{\sqrt{\tau_p}}\lambda^i, \quad \text{and} \quad A_\alpha=\frac{1}{2\pi\alpha'\sqrt{\tau_p}}a_\alpha .$$

Comparison of the amplitudes on the string and field theory sides fixes the normalization constant of the string amplitudes in terms of τ_p and k. At order $O(\alpha')$ there are five possible terms that can contribute to graviton scattering:

$$S^{(1)}_{\text{brane}}=-\frac{1}{2k_p^2}\int d^{p+1}\sigma\sqrt{-\tilde{G}}\{\beta_0\tilde{R}+\beta_1K^i_{\alpha\beta}K_i^{\alpha\beta} + \beta_2K^{i\alpha}_{\alpha}K^\beta_{i\beta}+\beta_3R_{\mu\nu}\perp^{\mu\nu}+\beta_4R_{\mu\nu\rho\sigma}\perp^{\mu\rho}\perp^{\nu\sigma}\}. \tag{22}$$

For the geometry of the submanifold we closely follow [6]. Denote by n_i^μ some orthonormal basis of normal vectors to the submanifold Σ representing the embedded p-brane. One can define the projection operator

$$\perp^{\mu\nu}=\sum_{i=p+1}^{D-1}n_i^\mu n_i^\nu=G^{\mu\nu}-\tilde{G}^{\mu\nu}, \tag{23}$$

where

$$\tilde{G}^{\mu\nu}=\frac{\partial X^\mu}{\partial\sigma^\alpha}\frac{\partial X^\nu}{\partial\sigma^\beta}\tilde{G}^{\alpha\beta}. \tag{24}$$

We need to know the expressions for the five terms in the action (22). $R_{\mu\nu\rho\sigma}$ and $R_{\mu\nu}$ are the usual Riemann and

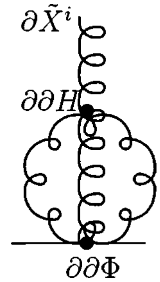

FIG. 2. Two-loop diagram with nonzero $\ln\Lambda$ contribution.

Ricci tensors. $\tilde{R}$ is the scalar curvature computed using the induced metric. $K^i_{\alpha\beta}$ is the second fundamental form defined by

$$K^i_{\alpha\beta}=\left(\frac{\partial^2X^\mu}{\partial\sigma^\alpha\partial\sigma^\beta}+\frac{\partial X^\nu}{\partial\sigma^\alpha}\frac{\partial X^\rho}{\partial\sigma^\beta}\Gamma^\mu_{\nu\rho}\right)n^i_\mu . \tag{25}$$

One also needs the bulk action to $O(\alpha')$ order. In [6], $S^{(1)}_{R^2\text{bulk}}$ was chosen in the Gauss-Bonnet form:

$$S^{(1)}_{R^2\text{bulk}}=\frac{1}{2k^2}\int d^Dx\frac{\alpha'}{4}e^{\gamma\Phi}(R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}-4R^{\mu\nu}R_{\mu\nu}+R^2). \tag{26}$$

Note that each of the above actions (22) and (26) is affected by field redefinitions [6]. As a result, the coefficients β_0, β_1, β_2, β_3, and β_4 are dependent on two parameters. Comparison of the amplitudes determines the coefficients:

$$\beta_0=1+\alpha, \quad \beta_1=-1+\alpha, \quad \beta_2=1-\alpha, \quad \beta_3=\beta,$$

$$\beta_4=-\alpha, \quad \text{and} \quad \frac{1}{2k_P}=\tau_p .$$

The choice of $S^{(1)}_{R^2\text{bulk}}$ in the Gauss-Bonnet form implies that the free parameters α and β are actually fixed: $\alpha=\beta=0$

Let us see how this result can be rederived from the σ-model perspective. Since this is a two-loop computation, we also need to take into account the one-loop beta function for the G field as well as the one-loop equations of motion. At one loop,

$$\beta^{G(1)}_{\mu\nu}=-\alpha' R_{\mu\nu}+O(\alpha'^2). \tag{27}$$

To simplify the computation we may assume that $H_{\mu\nu}$ depends only on σ^α and $H_{i\alpha}=0$. As in the previous section, expand all five terms from Eq. (22) and the σ-model actions (6) and (7) to linear order in $H_{\mu\nu}$ and quadratic order in $\tilde{\Phi}^i$. It turns out that only one two-loop diagram gives a contribution proportional to $\ln\Lambda$ (Fig. 2). It has the following structure: $\partial^\alpha\partial^\beta H_{ij}\partial_\alpha\partial_\beta\tilde{\Phi}^j$. This means that no other contractions between derivatives of $H_{\mu\nu}$ and derivatives of $\tilde{\Phi}^i$ modulo $R_{\mu\nu}$ could appear as a result of variation of Eq. (22). Performing the variation of Eq. (22) with respect to $\tilde{\Phi}^i$ explicitly gives an overdetermined system of equations on the coefficients β_0, β_1, β_2, β_3, and β_4, which reduces to $\beta_1=-\beta_2=0$, $\beta_0=-2\beta_4$, and β_3 is undetermined. Computing

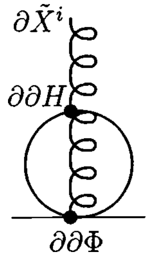

FIG. 3. Two-loop diagram with fermions with nonzero ln Λ contribution.

the diagram in Fig. 2 fixes the normalization. Altogether, we find $\beta_0=2$, $\beta_1=\beta_2=0$, β_3 undetermined, $\beta_4=-1$, and $1/k_p=2\tau_p$. This corresponds to the choice of $\alpha=1$ and is in agreement with the result in [6]. As was pointed out in [9], field redefinitions on the effective action side correspond to different choices of the renormalization schemes for σ-model computations. Thus in our scheme we would not get the $O(\alpha')$ corrections to the bulk action in the Gauss-Bonnet form. A nontrivial check of the above computation is the cancellation of all ln Λ terms in the beta function.

What happens in the case of the superstring? It is easy to check that diagrams containing fermion lines do not contribute at one loop. Thus the $O((\alpha')^0)$ part of the brane action is not affected. At two loops only one diagram (Fig. 3) gives a contribution proportional to ln Λ. In fact, it precisely cancels the contribution of the corresponding bosonic diagram (Fig. 2). Thus all coefficients β_0, β_1, β_2, and β_4 are zero. This means that there are no corrections to the BI action linear in curvature in the superstring case.

IV. σ-MODEL ANALYSIS OF A SYSTEM OF COINCIDENT D-BRANES

The σ-model analysis of a stack of coincident Dp-branes is more complicated. Instead of using $S_{\partial\Sigma}$ [Eq. (7)], we should rather be using a gauge invariant Wilson loop:

$$\tilde{S}_{\partial\Sigma}=-\log \mathrm{tr}P\exp\left\{i\oint_{\partial\Sigma}d\theta\left(A_\alpha\partial_\theta\sigma^\alpha+i\frac{1}{2\pi\alpha'}\Phi_i\partial_r X^i\right)\right\}. \tag{28}$$

Computations using the expansion of Wilson loops $\mathrm{tr}P\exp(i\int_{\partial\Sigma}A_\mu dX^\mu)$ are straightforward (see, for example, [10,11]). There is, however, a way to avoid it by introducing the auxiliary boundary fermions [12,8]. Now the Wilson loop can be rewritten as a functional integral over the boundary fermions coupled to the fields A_α and Φ_i via

$$e^{-S_{\partial\Sigma}(A,\Phi)}=\sum_{k=1}^{N}[d\lambda^\dagger d\lambda]\exp\left\{i\frac{2\pi k}{N}\left[\lambda^\dagger\lambda(\tau=\tau_0)+\frac{N}{2}-1\right]\right\}\exp\left[-\int_0^{2\pi}d\theta\,\lambda^\dagger\times\left(\frac{d}{d\theta}+\frac{1}{2\pi\alpha'}\partial_r X^i\Phi_i-i\partial_\theta\sigma^\alpha A_\alpha\right)\lambda\right]. \tag{29}$$

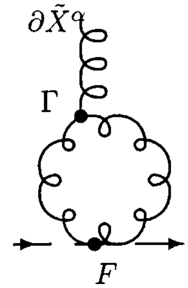

FIG. 4. Example of one-loop diagram with boundary fermions (represented by dashed lines).

Thus we have new auxiliary fermionic fields with propagators

$$\langle\lambda_a(\theta)\lambda_b^\dagger(\theta')\rangle=\frac{i}{2}\delta_{ab}\,\mathrm{sgn}(\theta-\theta'). \tag{30}$$

The boundary fermions couple to the string coordinates X^μ via $N\times N$ Hermitian traceless matrices A_α and Φ_i viewed as background fields.

In [8] this model was used to study world-volume potentials on a stack of coincident D-branes and world-volume couplings of NS fluxes, which are responsible for Myers' dielectric effect [13]. In this section we want to study the effect of the introduction of a nontrivial embedding of a stack of coincident D-branes in a curved target space.

As a first example, let us rederive some of the results of [10] in this model as opposed to the expansion of the Wilson loop. The one-loop beta function for the non-Abelian gauge field A_α was shown to be proportional to $(D^{A+\Gamma})^\beta F_{\alpha\beta}$. $D^{A+\Gamma}$ is a covariant derivative constructed using the gauge field and the Levi-Cività connection. This is simply the equations for the gauge field A_α in the background gravitational field. In order to compute the beta function for A_α, we need to consider renormalization of the coupling

$$-i\int d\theta\,\lambda_a^\dagger(A_\alpha)^{ab}\lambda_b\partial_\theta X^\alpha. \tag{31}$$

Expansion of this term is similar to Eq. (7) since the boundary fermions λ are quantum fields from the beginning. To first order in $H_{\alpha\beta}$, we get

$$-i\int d\theta\,\lambda^\dagger\left[\partial_\theta\tilde{X}^\alpha\left(A_\alpha+\partial_\beta A_\alpha\xi^\beta+\frac{1}{2}(\partial_\beta\partial_\gamma A_\alpha-\Gamma^\delta_{\beta\gamma}\partial_\delta A_\alpha-\partial_\alpha\Gamma^\delta_{\beta\gamma}A_\delta)\xi^\beta\xi^\gamma\right)+\partial_\theta\xi^\alpha(A_\alpha+(\partial_\beta A_\alpha-\Gamma^\gamma_{\alpha\beta}A_\gamma)\xi^\beta)\right]\lambda. \tag{32}$$

However, in order to make comparison easier, it is better to write it in the form

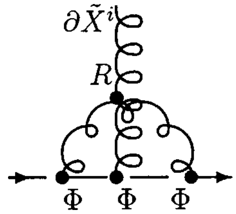

FIG. 5. Two-loop diagram contributing to $\mathrm{Tr}R_{ijkl}[\Phi^i\Phi^j][\Phi^k\Phi^l]$.

$$-i\int d\theta\Bigg\{\lambda_a^\dagger\Bigg(\partial_\theta\tilde{X}^\alpha A_\alpha-\partial_\theta\tilde{X}^\alpha\xi^\beta\tilde{F}_{\alpha\beta}-\frac{1}{2}\partial_\theta\tilde{X}^\alpha\xi^\beta\xi^\gamma\partial_\gamma\tilde{F}_{\alpha\beta}-\frac{1}{2}\partial_\theta\xi^\alpha\xi^\beta\tilde{F}_{\alpha\beta}+\frac{1}{2}\partial_\theta\tilde{X}^\alpha\xi^\gamma\xi^\delta\Gamma^\beta_{\gamma\delta}\tilde{F}_{\alpha\beta}\Bigg)^{ab}\lambda_b-\Bigg(A_\alpha\xi^\alpha+\frac{1}{2}\partial_\beta A_\alpha\xi^\alpha\xi^\beta-\frac{1}{2}\Gamma^\alpha_{\beta\gamma}A_\alpha\xi^\beta\xi^\gamma\Bigg)^{ab}\partial_\theta(\lambda_a^\dagger\lambda_b)\Bigg\}. \tag{33}$$

One can get it by using integration by parts along the lines of [14] (the authors of [14] considered an Abelian gauge field in a flat background). Here $\tilde{F}_{\alpha\beta}=\partial_\alpha A_\beta-\partial_\beta A_\alpha$. Now it is easy to see which diagrams will contribute to the corresponding terms in $D^{(A+\Gamma)}F$. For example, consider the diagram involving the bulk vertex shown in Fig. 4. Together with the diagram with the complex conjugate bulk vertex, it combines to give $-i\alpha'\Gamma_{\beta,\alpha\gamma}\partial_\theta\tilde{X}^\alpha F^{\beta\gamma}\ln\Lambda$. At this order,

$$\delta\Bigg(-\tau_p\mathrm{Tr}\int d^{p+1}\sigma\sqrt{-\det(\tilde{G}+F)}\Bigg)=\frac{\tau_p}{\alpha'}\int d^{p+1}\sigma\sqrt{-\tilde{G}}\beta^A{}_\alpha\delta A^\alpha. \tag{34}$$

When there are several vertices on the boundary one may worry that there will be many regions of integration corresponding to the relative positions of the angles on the boundary. However, as was shown in [8] using the symmetry properties of the ξ propagators on the boundary and the fact that they are double periodic functions of angles, it is possible to significantly reduce the number of regions. The perturbation theory becomes path ordered as in ordinary quantum mechanics. In this case the positions of ordered vertices lie in the interval $[\theta_i,\theta_f]\subset[0,2\pi]$.

V. $R\Phi^4$ CORRECTIONS TO THE EFFECTIVE ACTION

It is clear that this model provides a simple way of computing both world-volume potentials and derivative corrections to the brane action. The $O(\alpha')$ corrections to the effective action (22) have an overall trace in the non-Abelian case. Otherwise, the analysis of Sec. III holds in this case as well since it involves terms quadratic in Φ^i. Let us study another possible class of terms that could appear in the effective action: $R\Phi^4$. Consider two of them:

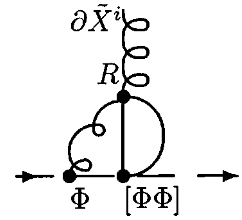

FIG. 6. Two-loop diagram with fermions contributing to $\mathrm{Tr}R_{ijkl}[\Phi^i\Phi^j][\Phi^k\Phi^l]$.

$$\int d\sigma^{p+1}\mathrm{Tr}(aR_{ijkl}[\tilde{\Phi}^i\tilde{\Phi}^j][\tilde{\Phi}^k\tilde{\Phi}^l]+bR_{ijkl}\tilde{\Phi}^i\tilde{\Phi}^k\tilde{\Phi}^j\tilde{\Phi}^l). \tag{35}$$

Using the cyclic property of the trace and the symmetries of R_{ijkl}, it is easy to see that for R_{ijkl} independent of $\tilde{\Phi}^i$ the second term is zero. Thus b cannot be determined at two loops. In order to determine the first coefficient we have to evaluate certain two-loop diagrams. In the bosonic case at order $O(H)$ the relevant diagram is shown in Fig. 5. To compute the diagram we will have to evaluate the following integral for different orderings of $\{\theta_1,\theta_2,\theta_3\}$:

$$\int\frac{d^2z\,d\theta_1 d\theta_2 d\theta_3(1-r^2)^2}{u_3(z-u_1)(\bar{z}-1/u_1)(z-u_2)(\bar{z}-1/u_2)(\bar{z}-1/u_3)^2}. \tag{36}$$

Here $u_i=e^{i\theta_i}$. We rewrite $\int d^2z=\int r\,dr\oint dw/iw$, $z=rw$, and $\bar{z}=r/w$. First, integrations over θ_1 and θ_3 are performed, then contour integration over w, and finally integration over r, introducing the cutoff. One integration over θ is left out since we want to find the renormalization of the $(1/2\pi\alpha')\int d\theta\,\partial_r\tilde{X}^i\Phi_i$ coupling. A somewhat lengthy but straightforward computation leads to the following answer for the sum of this diagram and the diagram with the complex conjugate bulk vertex:

$$\frac{-1}{(2\pi\alpha')^3}\frac{\alpha'^2}{4}(-\ln\Lambda)\int d\theta\,\partial\tilde{X}^i R_{ijkl}[\Phi^j[\Phi^k\Phi^l]] \tag{37}$$

Interpreting (37) as equations of motion, we find $a=-[\alpha'\tau_p/16(2\pi\alpha')^2]$.

In the case of a stack of D-branes there is an additional term containing fermions:

$$\int_{\partial\Sigma}d\theta(-\alpha'\psi^i\psi^j[\Phi_i\Phi_j]). \tag{38}$$

The diagram in Fig. 6 is the only two-loop diagram involving fermions proportional to $R\Phi^3$. It cancels the contribution of the corresponding bosonic diagram. Thus, $R\Phi^4$ corrections are absent in this case.

VI. DISCUSSION

In this paper we used a two-loop sigma model computation to determine certain gravitational corrections to the

D-brane action. In the bosonic case the α' corrections (22) were in agreement with those found in [6]. Corrections depending on the dilation and the B field will be the subject of future work. Interesting results in this direction using different techniques were obtained in [15]. In the superstring case we analyzed the possibility of $(\alpha')^2$ corrections of the form (35). Those terms could be of interest in AdS conformal field theory (CFT) correspondence or dynamics of giant gravitons.

For example, in the case of spaces of constant curvature with curvature independent of the transverse coordinates, Eq. (35) is proportional to $\mathrm{Tr}([\Phi^i,\Phi^j]^2)$, which is important for Myers' dielectric effect [13]. Thus it could be considered as a next order correction since the effective expansion parameter in the sigma model is α'/R_c^2. However, in the present paper we showed that those corrections are absent in the superstring case.

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